

# Multiparty-controlled remote preparation of four-qubit cluster-type entangled states

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We present a strategy for implementing multiparty-controlled remote state preparation (MCRSP) for a family of four-qubit cluster-type states with genuine entanglements while employing, Greenberg-Horne-Zeilinger-class states as quantum channels. In this scenario, the encoded information is transmitted from the sender to a spatially separated receiver via the control of multi-party. Predicated on the collaboration of all participants, the desired state can be entirely restored within the receiver's place with high success probability by application of appropriate local operations and necessary classical communication. Moreover, this proposal for MCRSP can be faithfully achieved with unit total success probability when the quantum channels are distilled to maximally entangled ones.

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## I. INTRODUCTION

An important focus in the field of quantum information processing (QIP) has been the secure and faithful transmission of information from one node of quantum network to another non-local node with finite classical and quantum resources. Quantum teleportation (QT) originated from the pioneering work of Bennett [1] is one application of non-local physics which may accomplish such a task. the central idea of QT is to deliver magically quantum information without physically transporting any particles from the sender to the receiver by means of an established entanglement. Apart from QT there exists another such efficient method, the so-called remote state preparation (RSP) [2–4]. RSP allows for the transfer of arbitrary known quantum states from a sender (Alice) to a spatially distant receiver (Bob), provided that the two parties share an entangled state and may communicate classically. Although both QT and RSP are able to achieve the task of information transfer [5–7], there are some subtle differences between QT and RSP which can be summarized as follow: (i) Precondition. In RSP, the sender of the states is required to be completely knowledge about the prepared state. In contrast, neither the sender nor the receiver necessarily possesses any knowledge of the information associated with the teleported states in QT. (ii) State existence. The state to be teleported initially inhabits a physical particle in the context of QT, while this is not required in RSP. That is to say, the sender in RSP is full aware of the information regarding the desired state, without any particle in such a state within his possession. (iii) Resource trade-off. Bennett [4] has shown that quantum and classical resources can be traded off in RSP but cannot in QT. In standard teleportation, an unknown quantum state is sent via a quantum channel, involving 1 ebit, and 2 cbits for communication. In contrast, if the teleported state is known to the sender prior to teleportation, the required resources can be reduced to 1 ebit and 1 cbit in RSP at the expense of lower

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success probability, half of that in QT. However, Pati [3] has argued that for special ensemble states (e.g., states on either the equator or great polar circle of the Bloch sphere) RSP requires less classical information than teleportation with the same unitary success probability.

Owing to its importance in QIP, RSP has received great attention and a large number of theoretical investigations have been proposed [8–38]. Specifically, there have been investigations concerning: low-entanglement RSP [8], optimal RSP [9], oblivious RSP [10, 11], RSP without oblivious conditions [12], generalized RSP [13], faithful RSP [14], joint RSP (JRSP) [15–27], Multi-controlled joint RSP [28], RSP for many-particle states [29–35], RSP for qutrit states [36] and continuous variable RSP in phase space [37, 38]. While, several RSP proposals by means of different physical systems have been experimentally demonstrated as well [39–45]. For examples, Peng *et al.* investigated a RSP scheme using NMR [39], Xiang *et al.* [40] and Peters *et al.* [41] proposed other two RSP schemes using spontaneous parametric down-conversion. Julio *et al.* [45] reported the remote preparation of two-qubit hybrid entangled states, including a family of vector-polarization beams; where single-photon states are encoded in the photon spin and orbital angular momentum, and then the desired state is reconstructed by means of spin-orbit state tomography and transverse polarization tomography.

Recently, many authors proceed to focus on RSP for cluster-type state by exploring various novel methods [46–51]; because cluster states are one of the most important resources in quantum information processing and can be efficiently applied to information processing tasks, such as: quantum teleportation [52], quantum dense coding [53, 54], quantum secret sharing [55], quantum computation [56], and quantum correction [57]. In general, a cluster-state is expressed as

$$|\Omega_N\rangle = \frac{1}{2^{N/2}} \bigotimes_{s=1}^N (|0\rangle_s Z_{(s+1)} + |1\rangle_s), \quad (1)$$

with the conventional use of  $Z$  is a pauli operator and  $Z_{N+1} \equiv 1$ . It has been shown that one-dimensional  $N$ -qubit cluster states are generated in arrays of  $N$  qubits mediated with an Ising-type interaction. It may easily be seen that the state will be reduced into a Bell state for  $N = 2$  (or 3); the cluster states are equivalent to Bell states (or Greenberger-Horne-Zeilinger (GHZ) states) respectively under stochastic local operation and classical communication (LOCC). Yet when  $N > 3$ , the cluster state and the  $N$ -qubit GHZ state cannot be converted to each other by LOCC. When  $N = 4$ , the four-qubit cluster-state is given by

$$|\Omega_4\rangle = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle). \quad (2)$$

In this work our aim is to examine the implementations of multiparty-controlled remote state preparation (MCRSP) for a family of four-qubit cluster-type entangled states with the aid of general quantum channels [46–51].

The paper is structured as follows: in the next section, we present the MCRSP scheme for four-qubit cluster-type entangled states with multi-agent control by the utilization of GHZ-class entanglements as quantum channels. The results show that the desired state can be faithfully reconstructed within Bob's laboratory with high success probability. Moreover, the required classical communication cost (CCC) and total success probability (TSP) will be discussed. Finally, features of our proposed scheme are detailed followed by a conclusion section.

## II. MCRSP FOR FOUR-QUBIT CLUSTER-TYPE ENTANGLED STATES

Suppose there are  $(m + n + 2)$  authorized participants, say, Alice, Bob, Charlie<sub>1</sub>,  $\dots$ , Charlie <sub>$n$</sub> , Dick<sub>1</sub>,  $\dots$ , and Dick <sub>$m$</sub>  (where  $m, n \geq 1$ ). To be explicit, Alice is the sender of the desired state, Bob is the receiver, and Charlie <sub>$i$</sub>  and Dick <sub>$j$</sub>  are truthful agents. Now, Alice would like to assist Bob remotely in the preparation of a four-qubit cluster-type entangled state

$$|P\rangle = \alpha|0000\rangle + \beta e^{i\varphi_0}|0011\rangle + \gamma e^{i\varphi_1}|1100\rangle + \delta e^{i\varphi_2}|1111\rangle, \quad (3)$$

with the control of the agents, where  $\alpha, \beta, \gamma, \delta$  and  $\varphi_i$  are real-valued, satisfy the normalized condition  $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ , and  $\varphi_i \in [0, 2\pi]$ . In order to obtain MCRSP, Alice, Bob, Charlie<sub>*i*</sub> and Dick<sub>*j*</sub> share previously generated genuine quantum resources – i.e., GHZ entanglements – which are given by

$$|\Upsilon^{(1)}\rangle_{A_1 A_2 B_1 B_2 C_1 \dots C_n} = \sum_k^{0,1} a_k |k\rangle_{A_1 A_2 B_1 B_2 C_1 \dots C_n}^{\otimes(n+4)}, \quad (4)$$

and

$$|\Upsilon^{(2)}\rangle_{A_3 A_4 B_3 B_4 D_1 \dots D_m} = \sum_l^{0,1} b_l |l\rangle_{A_3 A_4 B_3 B_4 D_1 \dots D_m}^{\otimes(m+4)}, \quad (5)$$

respectively, without loss of generality  $a_1, b_1 \in \mathbb{R}$ , and these bounds  $|a_0| \geq |a_1|$  and  $|b_0| \geq |b_1|$  are maintained. Initially, qubits  $A_1, A_2, A_3$  and  $A_4$  are sent to Alice, qubits  $B_1, B_2, B_3$  and  $B_4$  to Bob,  $C_i$  to Charlie<sub>*i*</sub> ( $i \in \{1, \dots, n\}$ ) and  $D_j$  to Dick<sub>*j*</sub> ( $j \in \{1, \dots, m\}$ ).

For implementing MCRSP, the procedure can be divided into the following steps:

**Step 1.** Firstly, Alice makes a two-qubit projective measurement on her qubit pair  $(A_1, A_3)$  under a set of complete orthogonal basis vectors  $\{|\mathcal{L}_{ij}\rangle\}$  composed of computational basis  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ , which can be written as

$$(|\mathcal{L}_{00}\rangle, |\mathcal{L}_{01}\rangle, |\mathcal{L}_{10}\rangle, |\mathcal{L}_{11}\rangle)^T = \mathcal{Q}(|00\rangle, |01\rangle, |10\rangle, |11\rangle)^T, \quad (6)$$

where,

$$\mathcal{Q} = \begin{pmatrix} \alpha & \beta e^{-i\varphi_0} & \gamma e^{-i\varphi_1} & \delta e^{-i\varphi_2} \\ \beta & -\alpha e^{-i\varphi_0} & \delta e^{-i\varphi_1} & -\gamma e^{-i\varphi_2} \\ \gamma & -\delta e^{-i\varphi_0} & -\alpha e^{-i\varphi_1} & \beta e^{-i\varphi_2} \\ \delta & \gamma e^{-i\varphi_0} & -\beta e^{-i\varphi_1} & -\alpha e^{-i\varphi_2} \end{pmatrix}. \quad (7)$$

Since the total systemic state taken as quantum channels can be described as

$$\begin{aligned} |\Psi_T\rangle &= |\Upsilon^{(1)}\rangle_{A_1 A_2 B_1 B_2 C_1 \dots C_n} \otimes |\Upsilon^{(2)}\rangle_{A_3 A_4 B_3 B_4 D_1 \dots D_m} \\ &= \sum_{i,j}^{0,1} |\mathcal{L}_{ij}\rangle_{A_1 A_3} \otimes |\mathcal{X}_{ij}\rangle_{A_2 A_4 B_1 B_2 B_3 B_4 C_1 \dots C_n D_1 \dots D_m}, \end{aligned} \quad (8)$$

where the non-normalized state  $|\mathcal{X}_{ij}\rangle \equiv {}_{A_1 A_3} \langle \mathcal{L}_{ij} | \Psi_T \rangle$  ( $i, j = 0, 1$ ) is obtained with probability of  $1/\mathcal{N}_{ij}^2$ , where  $\mathcal{N}_{ij}$  corresponds to the normalized parameter of state  $|\mathcal{X}_{ij}\rangle$ .

**Step 2.** According to her own measurement outcome  $|\mathcal{L}_{ij}\rangle$ , Alice makes an appropriate unitary operation  $\hat{U}_{A_2 A_4}^{(ij)}$  on her remaining qubit pair  $(A_2, A_4)$  under the ordering basis  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ , which is accordingly one of

$$\hat{U}_{A_2 A_4}^{(00)} = \text{diag}(1, 1, 1, 1), \quad (9)$$

$$\hat{U}_{A_2 A_4}^{(01)} = \text{diag}(e^{i\varphi_0}, -e^{-i\varphi_0}, e^{i(\varphi_2 - \varphi_1)}, -e^{i(\varphi_1 - \varphi_2)}), \quad (10)$$

$$\hat{U}_{A_2 A_4}^{(10)} = \text{diag}(e^{i\varphi_1}, -e^{i(\varphi_2 - \varphi_0)}, -e^{-i\varphi_1}, e^{i(\varphi_0 - \varphi_2)}), \quad (11)$$

and

$$\hat{U}_{A_2 A_4}^{(11)} = \text{diag}(e^{i\varphi_2}, e^{i(\varphi_1 - \varphi_0)}, -e^{i(\varphi_0 - \varphi_1)}, -e^{-i\varphi_2}). \quad (12)$$

Subsequently, Alice measures her qubits  $A_2$  and  $A_4$  under the a set of complete orthogonal basis vectors  $\{|\pm\rangle := \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)\}$ , and broadcasts her measured outcomes via a classical channel. Incidentally, all of the authorized

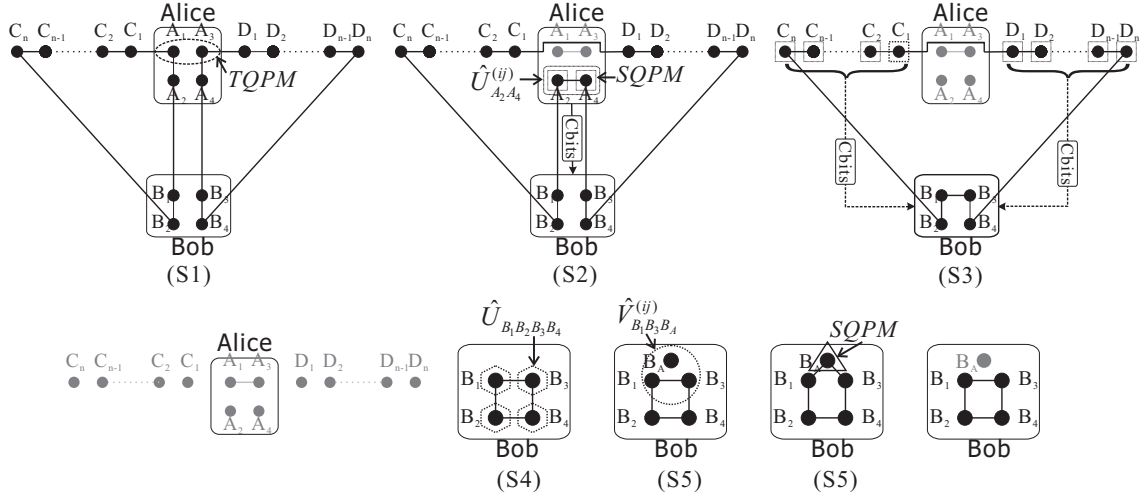


FIG. 1: Schematic diagram for MCRSP implementation. The procedure is explicitly decomposed as above Figures (S1)~(S5). The ellipse represents two-qubit projective measurement (TQPM) under the set of basis vectors  $\{|\mathcal{L}_{ij}\rangle\}$ ; the square represents single-qubit projective measurement (SQPM) under the set of basis vectors  $\{|\pm\rangle\}$ ; rectangle represents operating a bipartite collective unitary transformation  $\hat{U}_{A_2A_4}^{(ij)}$ ; the triangle represents SQPM under the set of basis vectors  $\{|0\rangle, |1\rangle\}$ ; the hexangle represents performing single-qubit unitary transformations  $\hat{U}_{B_1B_2B_3B_4}$  on Bob's qubits; the circle represents making a triplet collective unitary operation  $\hat{V}_{B_1B_3B_A}^{(ij)}$ ; Cbits represents classical information communication.

participants make an agreement in advance that cbits  $(i, j)$  correspond to the outcome  $|\mathcal{L}_{ij}\rangle_{A_1A_2}$ , and cbits  $(p, q)$  relate to the measuring outcome of qubits  $A_2$  and  $A_4$ , respectively. For simplicity, we denote

$$p, q = \begin{cases} 0, & \text{if } |+\rangle \text{ is probed} \\ 1, & \text{if } |-\rangle \text{ is probed} \end{cases}.$$

**Step 3.** The agents proceed to carry out single-qubit measurements under the set of vector basis  $\{|\pm\rangle\}$  on the qubits respectively, and later inform Bob of the results via classical channels. We assume that the cbits  $x_i$  corresponds to the outcome of the agents  $C_i$ , and  $y_j$  corresponds to the outcome of the agents  $D_j$ , where the values of  $x_i$  and  $y_j$  have been previously denoted as  $p$  and  $q$ , respectively. And we have  $g = \sum_{x=1}^n x_i \bmod \oplus 2$  and  $h = \sum_{y=1}^m y_j \bmod \oplus 2$ . Actually, there are four different situations, i.e., I)  $g = 0$  and  $h = 0$ ; II)  $g = 0$  and  $h = 1$ ; III)  $g = 1$  and  $h = 0$ ; and IV)  $g = 1$  and  $h = 1$ .

**Step 4.** In response to the different measuring outcomes of the sender and agents, Bob operates on his qubits  $B_1, B_2, B_3$  and  $B_4$  with an appropriate unitary transformation  $\hat{U}_{B_1B_2B_3B_4}$ .

**Step 5.** Finally, Bob introduces one auxiliary qubit  $B_A$  with initial state of  $|0\rangle$ . And then he makes triplet collective unitary transformation  $\hat{V}_{B_1B_3B_A}^{(ij)}$  on his qubits  $B_1, B_3$  and  $B_A$  under a set of ordering basis vector  $\{|000\rangle, |010\rangle, |100\rangle, |110\rangle, |001\rangle, |011\rangle, |101\rangle, |111\rangle\}$ , which is given by

$$\hat{V}_{B_1B_3B_A}^{(ij)} = \begin{pmatrix} \mathcal{W}_{ij} & \mathcal{U}_{ij} \\ \mathcal{U}_{ij} & -\mathcal{W}_{ij} \end{pmatrix}, \quad (13)$$

where  $\mathcal{W}_{ij}$  and  $\mathcal{U}_{ij}$  are  $4 \times 4$  matrices, respectively. To be explicit, we give

$$\mathcal{W}_{00} = \text{diag}\left(\frac{a_1b_1}{a_0b_0}, \frac{a_1}{a_0}, \frac{b_1}{b_0}, 1\right), \quad (14)$$

$$\mathcal{U}_{00} = \text{diag}\left(\sqrt{1 - \left(\frac{a_1b_1}{a_0b_0}\right)^2}, \sqrt{1 - \left(\frac{a_1}{a_0}\right)^2}, \sqrt{1 - \left(\frac{b_1}{b_0}\right)^2}, 0\right), \quad (15)$$

TABLE I:  $ijpqgh$  denotes the corresponding measurement outcomes from the authorized participants,  $\hat{U}_{B_1 B_2 B_3 B_4}$  denotes unitary operations what Bob needs to perform on qubits  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_4$ , respectively.

$ijpqgh$	$\hat{U}_{B_1 B_2 B_3 B_4}$	$ijpqgh$	$\hat{U}_{B_1 B_2 B_3 B_4}$	$ijpqgh$	$\hat{U}_{B_1 B_2 B_3 B_4}$	$ijpqgh$	$\hat{U}_{B_1 B_2 B_3 B_4}$
000000	$I_{B_1} I_{B_2} I_{B_3} I_{B_4}$	010000	$I_{B_1} I_{B_2} X_{B_3} X_{B_4}$	100000	$X_{B_1} X_{B_2} I_{B_3} I_{B_4}$	110000	$X_{B_1} X_{B_2} X_{B_3} X_{B_4}$
000001	$I_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	010001	$I_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	100001	$X_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	110001	$X_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
000010	$Z_{B_1} I_{B_2} I_{B_3} I_{B_4}$	010010	$Z_{B_1} I_{B_2} X_{B_3} X_{B_4}$	100010	$X_{B_1} Z_{B_1} X_{B_2} I_{B_3} I_{B_4}$	110010	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} X_{B_4}$
000011	$Z_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	010011	$Z_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	100011	$X_{B_1} Z_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	110011	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
000100	$I_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	010100	$I_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	100100	$X_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	110100	$X_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
000101	$I_{B_1} I_{B_2} I_{B_3} I_{B_4}$	010101	$I_{B_1} I_{B_2} X_{B_3} X_{B_4}$	100101	$X_{B_1} X_{B_2} I_{B_3} I_{B_4}$	110101	$X_{B_1} X_{B_2} X_{B_3} X_{B_4}$
000110	$Z_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	010110	$Z_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	100110	$X_{B_1} Z_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	110110	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
000111	$I_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	010111	$Z_{B_1} I_{B_2} X_{B_3} X_{B_4}$	100111	$X_{B_1} Z_{B_1} X_{B_2} I_{B_3} I_{B_4}$	110111	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} X_{B_4}$
001000	$Z_{B_1} I_{B_2} I_{B_3} I_{B_4}$	011000	$I_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	101000	$X_{B_1} Z_{B_1} X_{B_2} I_{B_3} I_{B_4}$	111000	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} X_{B_4}$
001001	$Z_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	011001	$Z_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	101001	$X_{B_1} Z_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	111001	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
001010	$I_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	011010	$I_{B_1} I_{B_2} X_{B_3} X_{B_4}$	101010	$X_{B_1} X_{B_2} I_{B_3} I_{B_4}$	111010	$X_{B_1} X_{B_2} X_{B_3} X_{B_4}$
001011	$I_{B_1} I_{B_2} I_{B_3} I_{B_4}$	011011	$I_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	101011	$X_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	111011	$X_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
001100	$Z_{B_1} I_{B_2} Z_{B_3} I_{B_4}$	011100	$Z_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	101100	$X_{B_1} Z_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	111100	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
001101	$Z_{B_1} I_{B_2} I_{B_3} I_{B_4}$	011101	$Z_{B_1} I_{B_2} X_{B_3} X_{B_4}$	101101	$X_{B_1} Z_{B_1} X_{B_2} I_{B_3} I_{B_4}$	111101	$X_{B_1} Z_{B_1} X_{B_2} X_{B_3} X_{B_4}$
001110	$I_{B_1} I_{B_2} I_{B_4}$	011110	$I_{B_1} I_{B_2} X_{B_3} Z_{B_3} X_{B_4}$	101110	$X_{B_1} X_{B_2} Z_{B_3} I_{B_4}$	111110	$X_{B_1} X_{B_2} X_{B_3} Z_{B_3} X_{B_4}$
001111	$I_{B_1} I_{B_2} I_{B_3} I_{B_4}$	011111	$I_{B_1} I_{B_2} X_{B_3} X_{B_4}$	101111	$X_{B_1} X_{B_2} I_{B_3} I_{B_4}$	111111	$X_{B_1} X_{B_2} X_{B_3} X_{B_4}$

$$\mathcal{W}_{01} = \text{diag}(\frac{a_1}{a_0}, \frac{a_1 b_1}{a_0 b_0}, 1, \frac{b_1}{b_0}), \quad (16)$$

$$\mathcal{U}_{01} = \text{diag}(\sqrt{1 - (\frac{a_1}{a_0})^2}, \sqrt{1 - (\frac{a_1 b_1}{a_0 b_0})^2}, 0, \sqrt{1 - (\frac{b_1}{b_0})^2}), \quad (17)$$

$$\mathcal{W}_{10} = \text{diag}(\frac{b_1}{b_0}, 1, \frac{a_1 b_1}{a_0 b_0}, \frac{a_1}{a_0}), \quad (18)$$

$$\mathcal{U}_{10} = \text{diag}(\sqrt{1 - |\frac{b_1}{b_0}|^2}, 0, \sqrt{1 - (\frac{a_1 b_1}{a_0 b_0})^2}, \sqrt{1 - (\frac{a_1}{a_0})^2}), \quad (19)$$

$$\mathcal{W}_{11} = \text{diag}(1, \frac{b_1}{b_0}, \frac{a_1}{a_0}, \frac{a_1 b_1}{a_0 b_0}), \quad (20)$$

and

$$\mathcal{U}_{11} = \text{diag}(0, \sqrt{1 - (\frac{b_1}{b_0})^2}, \sqrt{1 - (\frac{a_1}{a_0})^2}, \sqrt{1 - (\frac{a_1 b_1}{a_0 b_0})^2}). \quad (21)$$

Then, Bob measures his auxiliary qubit,  $B_A$ , under a set of measuring basis vectors  $\{|0\rangle, |1\rangle\}$ . If state  $|1\rangle$  is measured, his remaining qubits will collapse into the trivial state, and the MJRSP fails in this situation; otherwise,  $|0\rangle$  is probed, and the qubits' state will transform into the desired state, that is, our MCRSP is successful in this case.

Based on the above five-step protocol, it has been shown that the MCRSP for a family of cluster-type states can be faithfully performed with predictable probability. The steps can be decomposed into a schematic diagram shown in Fig. 1. As a summary, we list Bob's required local single-qubit transformations according to the sender's and agents' different measurement outcomes in Table I.

From the above analysis, one can see that the prepared state can be faithfully reconstructed with specified success probabilities.

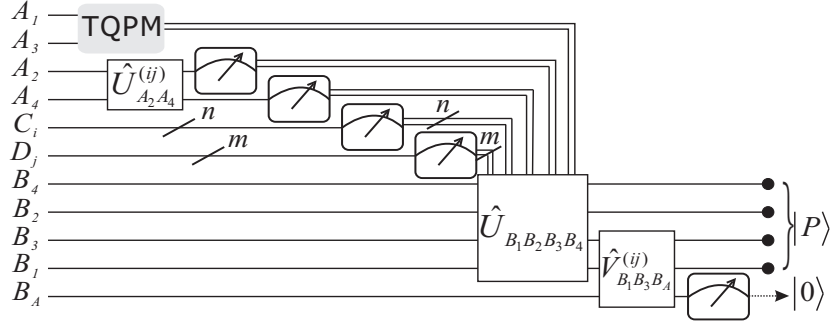


FIG. 2: Quantum circuit for implementing the MCRSP scheme. TQPM denotes two-qubit projective measurement under a set of complete orthogonal basis vectors  $\{|\mathcal{L}_{ij}\rangle\}$ ;  $\hat{U}_{A_2A_4}^{(ij)}$  denotes Alice's appropriate bipartite collective unitary transformation on qubit pair  $(A_2, A_4)$ ;  $\hat{U}_{B_1B_2B_3B_4}$  denotes Bob's appropriate single-qubit unitary transformations on his qubits  $B_1, B_2, B_3$  and  $B_4$ , respectively, and  $\hat{V}_{A_1A_3B_A}^{(ij)}$  denotes Bob's triplet collective unitary transformation on his qubits  $B_1, B_3$  and  $B_A$ .

Now, let us turn to calculate the TSP and CCC. Alice's measurement outcome,  $|\mathcal{L}_{ij}\rangle$ , has an occurrence probability of

$$P_{|\mathcal{L}_{ij}\rangle} = \frac{1}{\mathcal{N}_{ij}^2}. \quad (22)$$

Furthermore, in considering the capture of the state  $|0\rangle_{B_A}$ , the probability should be

$$P_{|0\rangle_{B_A}} = |\mathcal{N}_{ij}a_1b_1|^2. \quad (23)$$

Thus, the success probability of MCRSP for the measurement outcome  $(i, j)$  should be given by

$$P_{(i,j)} = P_{|\mathcal{L}_{ij}\rangle} \times P_{|0\rangle_{B_A}} = |a_1b_1|^2. \quad (24)$$

In terms of  $P_{(i,j)}$ , one can easily obtain that the TSP sums to

$$P_{\sum_{i,j}^{0,1}(i,j)} = \sum_{i,j}^{0,1} P_{(i,j)} = 4|a_1b_1|^2. \quad (25)$$

Moreover, one can show that the required CCC should be  $(2 + 2 + m + n) = (m + n + 4)$  cbits totally.

Herein, we had described our proposal of MCRSP for a family of four-qubit cluster-type entangled states. We have proved that our scheme can be realized faithfully with TSP of  $4|a_1b_1|^2$  and CCC of  $(m + n + 4)$  via the control of multi-agent in a quantum network. For clarity, the quantum circuit for our MCRSP protocol is displayed in Fig. 2.

### III. DISCUSSIONS

We have found several remarkable features with respect to the scheme presented above and these features are summarized as follows: (1) To the best of our knowledge, this is the first time one has exploited such a scenario concerning MCRSP for four-qubit cluster-type entangled states via control of  $(m + n)$ -party. Information conveyance only takes place between the sender and the receiver, i.e.,  $1 \rightarrow 1$  threshold communication. Moreover, the agents are capable of supervising and switching the procedure during the relay of information communication. Secure multi-node information communication is considerably important in prospective quantum networks. (2) Generally, our MCRSP can be faithfully performed with TSP of  $4|a_1b_1|^2$ . Moreover, when the state  $|a_1| = |b_1| = 1/\sqrt{2}$  is chosen in the beginning, thus the channels become maximally entangled, the TSP can reach unity as shown in Fig. 3.

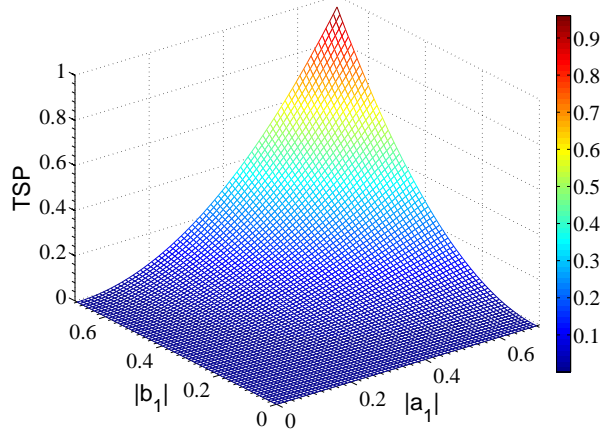


FIG. 3: The relation between TSP and the smaller coefficients of entanglements severed as quantum channels.

Consequently, that indicates our scheme becomes a deterministic one in this case. Additionally, it should be noted that the parameters  $a_1$  and  $b_1$  relate to the Shannon entropies of the employed quantum channels,

$$H(f) = -|f|^2 \log |f|^2 - (1 - |f|^2) \log(1 - |f|^2), \quad (26)$$

where  $f \in \{a_1, b_1\}$  and  $a_1, b_1 \in [-\sqrt{2}/2, \sqrt{2}/2]$ . The entropy will vary with the coefficients specific to different quantum channels depicted in Fig. 4. Note, the entropy in essence reflects inherent property (i.e., entanglements) of quantum channels. (3) Our scheme enables one to fulfill RSP via the multi-agent control. Incidentally, all of the agents are capable of switching the preparation procedures. The desired state can be recovered at Bob's site conditioned to the total collaboration of network members. Anyone of the party cannot recover the desired state by themselves. In this sense, the security of information is to a large extent guaranteed. (4) Within our scheme, there exists  $(m + n)$  controllers to manipulate or switch the preparation procedure. If both  $m$  and  $n$  are chosen to be 0, there are no authorized controllers during the process of the preparation, it has been found that our scheme is smoothly reduced to a scheme resembling RSP for four-qubit cluster-type states with TSP of  $4|a_1 b_1|^2$ . In this case, the measurements made by the controllers and the communication between controllers and receivers are unnecessary, as is the auxiliary qubit. Now, we can compare our reduced scheme with other previous schemes [46–51]; we do this with respect to RSP and JRSP for such states in view of the resource consumption and quantum operation complexity as shown in Table II.

From Table II, one can directly note that the TSP of our scheme is capable of unity, and the intrinsic efficiency ( $\eta$ ) achieves 33.33%, which is much greater than those in the previous schemes [46–51]. Due to characteristic high-efficiency and high-TSP in the present scheme, it is both highly efficient and optimal in comparison to the existed ones. Incidentally, the intrinsic efficiency of a scheme is defined by [58]

$$\eta = \frac{N_s}{N_q + N_c} \times TSP, \quad (27)$$

where  $N_s$  weights the amount of qubits of the prepared states,  $N_q$  weights the amount of quantum resource consumption, and  $N_c$  weights the amount of CCC in quantum computation. Additionally, Ref. [50] can be realized with a TSP of 100%; however, there are several crucial differences between our methods and the previous, they are as follows: (i) Quantum resource consumption. In [50], 12 qubits are indispensable in the course of RSP for four-qubit cluster-type states, while 8 qubits are sufficient to implement RSP for such states in our reduced scheme. Implying our scheme is more economic. (ii) Operation complexity. Two four-qubit projective measurements in [50] are required for their procedure, while two-qubit projection measurements are required in our scheme. Experimental realization

TABLE II: Comparison between our scheme and the previous ones in the case of maximally entangled channels. ET represents entanglement; SQ represents single-qubit; ASQ represents auxiliary single-qubit; CNOT represents controlled-not gates; PM represents projective measurement; SQPM represents single-qubit projective measurement and TSP represents total success probability.

Schemes	Required qubits	Quantum operations	CCC	TSP	$\eta$
Ref. [46]	six 2-qubit ETs	two 4-qubit PMs	8	$\frac{1}{16}$	1.25%
	two 6-qubit ETs	two 4-qubit PMs	8	$\frac{1}{16}$	1.25%
Ref. [47]	two 4-qubit ETs	two 2-qubit PMs	4	$\frac{1}{4}$	8.33%
Ref. [48]	two 3-qubit ETs & two ASQ	two 2-qubit PMs & 2 CNOTs	4	$\frac{1}{4}$	8.33%
Ref. [49]	two 2-qubit ETs & four ASQ	two 4-qubit PMs & 4 CNOTs	4	$\frac{1}{4}$	8.33%
Ref. [50]	six 2-qubit ETs	two 4-qubit PMs	8	1	20.00%
Ref. [51]	two 2-qubit & one 3-qubit ET	one 3-qubit PM	3	$\frac{1}{4}$	10.00%
Current scheme	two 4-qubit ETs	one 2-qubit PMs & two SQPMs	4	1	33.33%

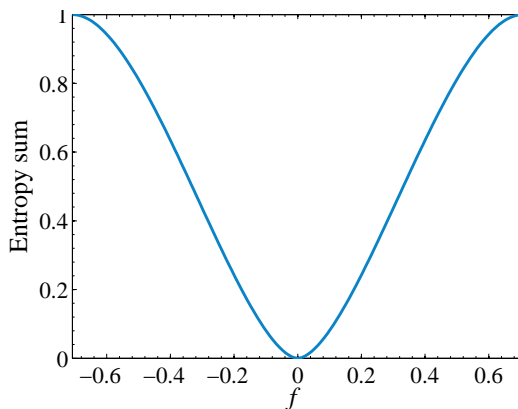


FIG. 4: The entropic diagram with variation of the parameter of quantum channels.

of four-qubit projective measurement is much more difficult than that for two-qubit. Thus, in principal our scheme is easier to experimentally realize than the previous method.

#### IV. CONCLUSION

Herein we have derived a novel strategy for implementing MCRSP scheme for a family of four-qubit cluster-type entangled states by taking advantage of robust GHZ-class states as quantum channels. With the aid of suitable LOCC, our scheme can be realized with high success probability. Remarkably, our scheme has several nontrivial features, including high success probability, security and reducibility. Particularly, the TSP of MCRSP can reach unity when the quantum channels are distilled to maximally entangled ones; that is, our scheme can be performed deterministically at this limit. We argue the current MCRSP proposal might open up a new way for long-distance communication in prospective multi-node quantum networks.



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